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How are Working Hours and Wage Earnings Determined? A Theoretical and Empirical Study

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Abstract: *Incorporating both supply and demand functions of working hours, we constructed a model of working hours. The model demonstrates that (1) working hours and wage earnings are determined jointly at an equilibrium point on a contract curve where the demand and supply of workers are equal, and (2) the contract curve passes through the intersection of the supply and demand curves of working hours. These results imply that (1) it is impossible to estimate the supply curve of working hours because equilibrium points are not usually located on a supply curve of working hours, and (2) a contract curve of working hours should be estimated to measure the wage elasticity of working hours.*

For the estimation of contract curves, geometric mean regression (GMR) was selected for three reasons. First, both variables in the contract curve equations are measured with errors. Second, the GMR estimator is the maximum likelihood estimator. Third, the coefficient of determination (R^2) of both variables is equal in the GMR. Applying GMR, the contract curves of working hours and their wage elasticity were estimated. The estimated wage elasticity of working hours was between -0.13 and -0.23.

Keywords: *working hours, wage rate, contract curve, geometric mean regression.*

JEL code: *J22, J23, C13.*

1. Introduction

The framework of the supply curve of working hours is a common tool to explain how working hours are determined or how working hours respond to tax rate changes. Thus, many empirical articles have been written on this topic. However, the estimation task is not yet complete. Keane (2011) and Bargain and Peichl (2013) argued that there is no clear consensus on the magnitude of the wage elasticity of the supply curve.[1]

Pencavel (2016) critically wrote on the basic assumption of the canonical model: "it has become orthodox in economics research to interpret the association between hourly earnings and working hours as the expression of the preferences of workers." In brief, this means that

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under a given wage rate, employees determine working hours and employers accept it, or the employer is indifferent on the length of working hours. Pencavel claims that this assumption is not realistic, and the hypothesis must be tested.

Pencavel's proposal is to posit another hypothesis: "working hours are determined at the intersection of demand and supply curves of working hours." Accordingly, he stresses the importance of identifying the supply curve of working hours.

As suggested by Pencavel (1986, 2016) this study constructed a model that incorporates both supply and demand functions of working hours. The constructed model demonstrates that working hours and wage earnings are determined at an equilibrium point on a contract curve. Consequently, we argue that a contract curve should be estimated.

In the estimation, we applied geometric mean regression (GMR), which is a special case of errors-in-variables regression or Deming regression. We chose this method because both variables in the contract curve equation are measured with errors.

The remainder of this paper is organised as follows: Section 2 presents a basic model. In Section 3, the GMR is explained. Then, by applying GMR, we estimate the contract curve of working hours and its wage elasticity. Finally, Section 4 presents our conclusion.

2. Basic model[2]

2.1. *Model assumptions*

The assumptions of the model are ordinary and are as follows:

(i) The worker utility function is U(E, h), where E and h denote wage earnings and working hours, respectively. U(E, h) is quasiconcave, and $U_E > 0$ and $U_h < 0$.

(ii) Firms have the production function AF(L, h), where L denotes the number of employees and A is total factor productivity. The capital stock is constant.

(iii) The equilibrium wage rate is determined to equate the demand and supply of workers.

2.2. *Supply function of working hours*

To clarify the model, three examples of utility function U(E, h) were used. Then, the derived supply curves of working hours were described in the h-E plane.

Case 1: $U(E, h) = E - \alpha(h + \beta)^2$, $(\alpha > 0, \beta > 0)$, $(E > 0, h > 0)$.

Here, α and β are positive parameters. $U(E, h)$ is quasiconcave in the defined domain $(E>0, h>0)$, and the indifference curves are parabolic. The utility-maximising behaviour of a worker is formulated as follows: Max $U(E, h) = E - \alpha(h + \beta)^2$, st. $E = wh$. Here, w is the wage rate. The first-order condition is $dU/dh = w - 2\alpha(h + \beta) = 0$, from which we have the following supply function of working hours: $w = 2\alpha(h + \beta)$. Multiplying both sides of the equation by h, we obtain the supply curve of working hours in the h-E plane as follows:

$$E = 2\alpha h(h + \beta)\ .\ (\alpha > 0, \beta > 0) \tag{1}$$

Case 2: $U(E, h) = -\alpha(E - \beta)^2 - h$, $(\alpha > 0, \beta > 0)$, $(\beta > E > 0, h > 0)$.

$U(E, h)$ is quasiconcave in the defined domain $(\beta > E > 0, h>0)$, and the indifference curves are parabolic. As in Case 1, the supply curve of working hours in the h-E plane is derived as follows:

$$h = -2\alpha E(E - \beta)\ .\ (\alpha > 0, \beta > 0) \tag{2}$$

The supply curve of working hours is a parabola and backward bending.

Case 3: $U(E, h) = E/(h + \alpha)^\beta$, $(\alpha > 0, \beta > 1)$, $(E > 0, h > 0)$.

The utility function is quasiconcave in the defined domain $(E > 0, h>0)$. As in Case 1, the supply curve of working hours in the h-E plane is derived as follows:

$$h = \alpha/(\beta - 1).\ (\alpha > 0, \beta > 1) \tag{3}$$

The supply curve of working hours is a vertical line.

2.3. Demand function of working hours

To clarify the model, a Cobb Douglas production function is assumed as follows:

$$AF(L, h) = AL^\gamma h^\delta,\ (1>\gamma>\delta> 0),$$

where γ and δ are positive parameters, and A is total factor productivity.

A demand function of working hours is derived from the profit maximising behaviour of the firm. First, an isoprofit curve is derived. Then using it, a demand function of working hours is derived. As explained in note (3), the isoprofit curves are derived as follows:

$$E = kh^{(\delta/\gamma)} - C \tag{4}$$

where C represents the fixed costs per employee, and k denotes the profit level. (If k decreases, the profit level increases.)[3]

Next, a demand curve of working hours is derived from a firm's profit maximising behaviour. It is formulated as follows: Min $k = (E + C)/ h^{(\delta/\gamma)}$ st. $E = wh$. From the first-order condition, the demand curve of working hours in the h-E plane is derived as follows:

$$E = C/(\gamma/\delta - 1). \ (\gamma/\delta>1) \tag{5}$$

The demand curve of working hours is a horizontal line.

2.4. Equilibrium of a firm and its workers

Figure 1 illustrates the demand and supply curves of working hours in a firm and its workers (the supply curve is from case 1). These two curves intersect at Q, where the wage rate is ∠QOM, and the desired working hours of a firm and its workers coincide.

Is equilibrium realised at Q (the intersection of supply and demand curves of working hours)? The condition for equilibrium is that the demand and supply of workers should be equal at the wage rate (Rosen 1969, p. 261). This is possible only when ∠QOM is equal to the market wage rate by chance. However, there is no assurance that this is true.

If the market wage rate is ∠ROM (> ∠QOM), there will be an excess demand for workers at Q. Then, the equilibrium point moves up along the contract curve, which is the locus of the tangency points of the indifference and isoprofit curves. (The contract curve passes through point Q.) Thus, R becomes the equilibrium point for the firm and its workers.

In summary, the equilibrium point lies somewhere on the contract curve where the demand and supply of workers are equal. We name the contract curve the wage–hour (WH) contract curve. The result implies that (1) it is impossible to estimate the supply curve of working hours because equilibrium points are not usually located on a supply curve of working hours, and (2) a contract curve of working hours should be estimated to measure the wage elasticity of working hours. In the example of Case 1, the WH contract curve is derived as follows: $(\delta/\gamma)(E + C) = 2\alpha h(h + \beta)$.[4]

2.5. Wage-hour contract curve of working hours

The equations of the WH contract curve in the three cases are calculated as follows. The contract curves are either positively sloped or negatively sloped.

	Supply function of working hours	*WH Contract curve*
Case 1	$E = 2\alpha h(h + \beta)$	$(\delta/\gamma)(E + C) = 2\alpha h(h + \beta)$.
Case 2	$h = -2\alpha E(E - \beta)$	$h = -2\alpha(\delta/\gamma)(E + C)(E - \beta)$
Case 3	$h = \alpha/(\beta - 1)$	$(1 + C/E)(1+ \alpha/h) = \beta/(\delta/\gamma)$

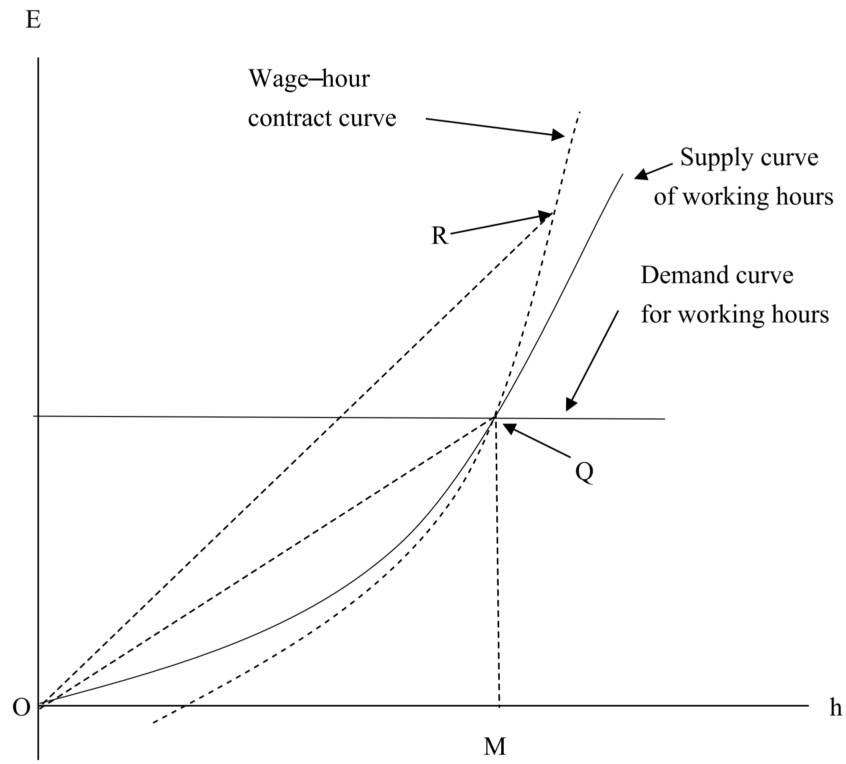

Figure 1: Equilibrium of a firm and its workers

In Case 1, the WH contract curve is a parabola and positively sloped ($dE/dh > 0$). In Case 2, the WH contract curve is a parabola and backward bending. In Case 3, it is a negatively sloped curve ($dE/dh < 0$). In summary, the WH contract curves are either positively or negatively sloped. Therefore, the wage elasticity of working hours is either positive or negative. It must be empirically measured.

2.6. Market equilibrium of an industry

How are working hours and wage earnings determined in market equilibrium? We considered an industry where all firms had the same production function (or isoprofit curves) and all workers had the same utility function (or indifference curves). The industry's market equilibrium was described by three endogenous variables (E, h, and L) and the following three equations:

$$AF_L(L, h) = E + C \tag{6-1}$$

$$L = L^s(E/h) \tag{6-2}$$

$$U_h(E, h)/U_E(E, h) = Y_h(E, h)/Y_E(E, h). \tag{6-3}$$

Equation (6-1) is the demand function for workers in implicit form, where AF(L, h) is the aggregate production function of the industry. Equation (6-2) is the supply function of workers, which is an increasing function of the wage rate (E/h). Equation (6-3) shows the WH contract curve in its general form. Here, $Y(E, h) = k_1$ and $U(E, h) = k_2$ are the representative firm's isoprofit curves and the representative worker's indifference curves, respectively (k_1 and k_2 are parameters).

By inserting Equation (6-2) into (6-1), we find that $AF_L(Ls(E/h), h) = E + C$, which is the equation of the workers' market equilibrium (WM equilibrium curve). As shown in Figure 2, this curve has a positive slope. Thus, the market equilibrium is at point R, which is the intersection of the WM equilibrium curve and the WH contract curve.[5]

If the supply of workers increases, the WM equilibrium curve shifts downward. Then, the equilibrium point moves downward from R to R′ along the WH contract curve, and the market wage rate will decrease.

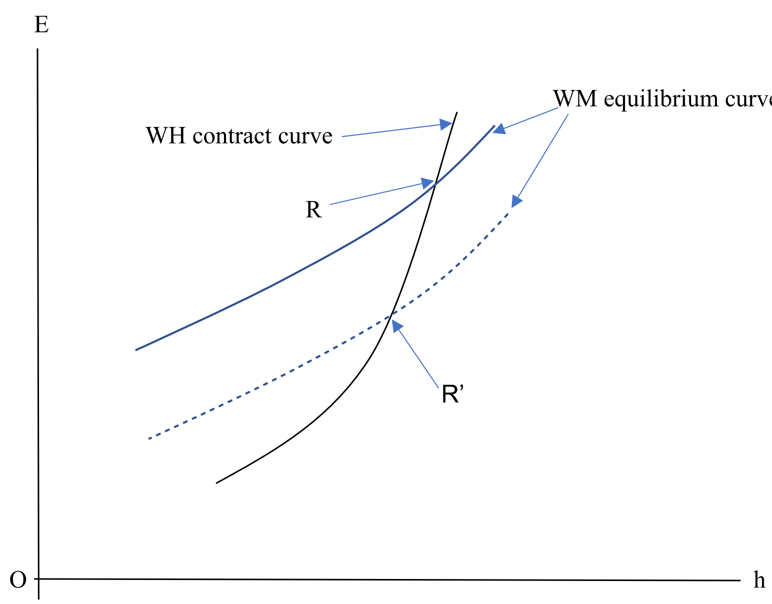

Figure 2: Market equilibrium of an industry

3. Estimation of contract curves of working hours

3.1. Strategy

We considered a group of manufacturing industries. Assumed that all industries had the same production function as AF(L, h) with various levels of A. Additionally, we assumed that all workers had the same preference (indifference curves). We then showed that the equilibrium point of each industry lied on a common WH contract curve.

Note (3) shows that the isoprofit curve of a firm whose production function is AF(L, h) is the solution E(h) of the differential equation. A (total factor productivity) did not appear in the differential equation. Therefore, industries (firms) having the same production function with various levels of A have the same isoprofit and demand curves for working hours. Thus, if their workers share the same preference, they have a common WH contract curve. The equilibrium point of an industry with a larger A is located at a higher point on the common WH contract curve.

In brief, we first took a group of industries that were supposed to have similar production functions. Second, we restricted the workers to a group that was supposed to have the same preference. Finally, using their data on wage earnings (E) and working hours (h), we can estimate the WH contract curve.

3.2. Data

We employed the Basic Survey on Wage Structure (BSWS), which is conducted every June by the Japanese Ministry of Health, Labour and Welfare. The BSWS is a type of industry survey. As employers responded to the questionnaire, we consider data on working hours and wage earnings to indicate equilibrium points on their WH contract curves.

The survey classified 90 industries and differentiated firms by their number of employees: (1) 1,000 or more employees, (2) 100–999 employees, and (3) 10–99 employees. Additionally, the survey indicated workers' educational background (i.e., college or high school graduates), gender (i.e., male or female) and age.

The scheduled working hours (SWH) and overtime working hours (OTH) were selected from the BSWS. The former are standard working hours, as stipulated by office regulations. Therefore, h = SWH+OTH is total working hours. For wage earnings (E), contractual cash earnings (CCE) and annual special cash earnings (ASE) are selected. The CCE includes payments for both SWH and OTH in June of the survey year. ASE is the previous year's annual bonus payments. Thus, we calculate the monthly wage earnings (E) as E = CCE+ASE/12.

Figure 3 presents plots of 14 manufacturing industries with male workers, college graduates, and those aged 50–54. We assumed that the industries had similar production functions and that their workers had similar preferences. The vertical axis denotes wage earnings (1,000 yen),

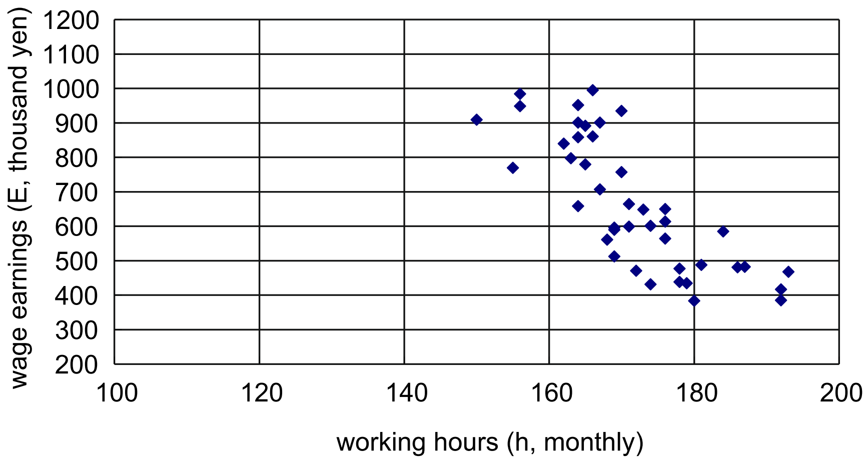

Figure 3: Working hours (h) and wage earnings (E) in fourteen manufacturing industries (male, college graduate, age 50–54, 2015)

and the horizontal axis denotes monthly working hours. We estimated the WH contract curve using a regression analysis of the plots.[6]

3.3. Geometric mean regression

Geometric mean regression (GMR) was used, which is a special case of errors-in-variables regression or Deming regression. We cannot use ordinary least squares (OLS) because both working hours (h) and wage earnings (E) are measured with errors.[7]

Errors-in-variables regression is explained as follows: In the contract curve model, $E = \alpha + \beta h$ is assumed to be a linear relationship between the two variables. The two observed variables (x_i, y_i), $i = 1 \ldots n$, have errors that may be written as follows:

$$x_i = h_i + e_{xi},$$

$$y_i = E_i + e_{yi},$$

where e_{xi} and e_{yi} are random variables. It is assumed that e_{xi}s are i.i.d. with $e_{xi} \sim N(0, \sigma^2)$ and that e_{yi}s are i.i.d. with $e_{yi} \sim N(0, k\sigma^2)$. The maximum likelihood estimators are then derived as follows:[8]

$$\hat{\beta} = \frac{S_{yy} - kS_{xx} + \sqrt{(S_{yy} - kS_{xx})^2 + k(2S_{xy})^2}}{2S_{xy}} \tag{7.1}$$

$$\hat{\alpha} = \overline{y} - \hat{\beta}\overline{x}, \tag{7.2}$$

$$\hat{h}_i = \frac{kx_i + \hat{\beta}(y_i - \hat{\alpha})}{k + \hat{\beta}^2}, \tag{7.3}$$

$$\hat{E}_i = \hat{\alpha} + \hat{\beta}\hat{h}_i, \tag{7.4}$$

where $S_{xx} = [1/(n-1)]\Sigma_{i=1}^{n}(x_i - \overline{x})^2$,

$S_{xy} = [1/(n-1)]\,\Sigma_{i=1}^{n}(x_i - \overline{x})(y_i - \overline{y})$,

$S_{yy} = [1/(n-1)]\Sigma_{i=1}^{n}(y_i - \overline{y})^2$.

Equation (7-1) for estimator $\hat{\beta}$ includes k (= var(ey_i)/var(ex_i)). The true value of k is unknown. If we assume that k is equal to the ratio of the sample variances (k = S_{yy}/ S_{xx}), then it reduces to the following:

$$\hat{\beta} = sign(S_{xy})\sqrt{S_{yy} / S_{xx}}. \tag{8}$$

This is known as the GMR estimator.[9]

Hereafter, in the estimation, we use the GMR from the perspective of R^2 (i.e., the coefficient of determination). In the errors-in-variables regression, if the same definition of R^2 as in OLS is applied, it is defined as $R_h^2 = \frac{\Sigma(\hat{h}_i - \overline{x})^2}{\Sigma(x_i - \overline{x})^2}$ and $R_E^2 = \frac{\Sigma(\hat{E}_i - \overline{y})^2}{\Sigma(y_i - \overline{y})^2}$ $(i = 1 \ldots n)$. In GMR, using k = S_{yy}/S_{xx}, we obtain $R_h^2 = R_E^2$. As the two variables (h and E) are treated symmetrically in the model, it is desirable that $R_h^2 = R_E^2$.[10]

Figure 4 presents the relationship between the observed value B(x_i, y_i) and its corresponding estimate $M(\hat{h}_i, \hat{E}_i)$ in the GMR. Let us then draw vertical and horizontal lines from B. We then let the intersections with the regression line be $C(x_i, \hat{\alpha} + \hat{\beta}x_i)$ and $D((y_i - \hat{\alpha})/\hat{\beta}, y_i)$, respectively. The point of estimate $M(\hat{h}_i, \hat{E}_i)$ is the midpoint of CD.[11]

Table 1 compares GMR with OLS regression using the data presented in Figure 3. There is a simple relationship between GMR and OLS regression. Column (1) presents the GMR estimate. Column (2) presents the OLS

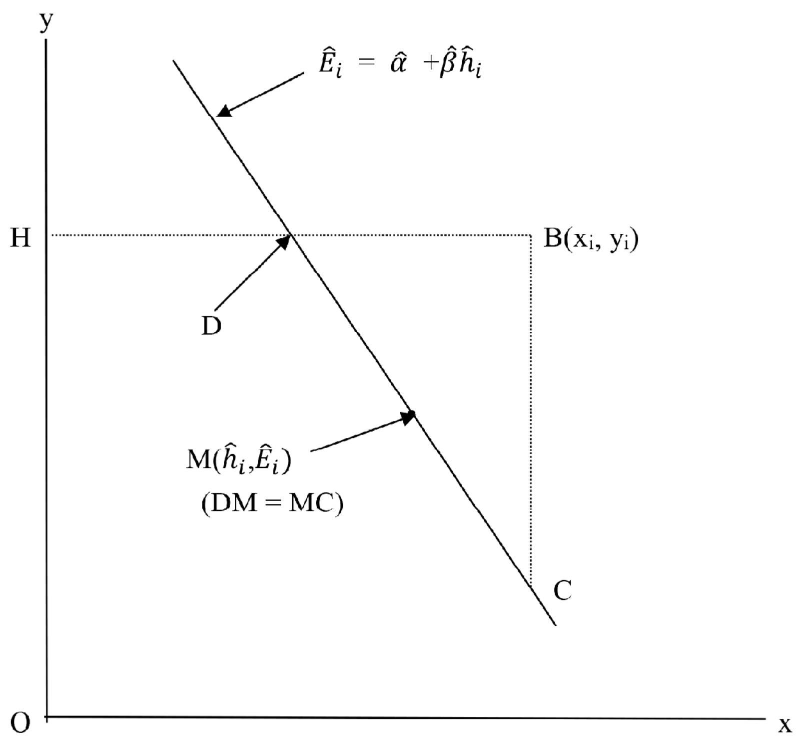

Figure 4: Observation B(x_i, y_i) and its estimate M($\hat{t}_i$, $\hat{E}_i$) in GMR

regression estimate of y on x, and Column (3) shows the OLS regression estimate of x on y. $\hat{\beta}$ of the GMR is the geometric mean of the two corresponding OLS estimates, or $(–19.02)^2 = (–15.04)/(?0.0416)$.[12]

Rows (iv) and (v) of Table 1 present the correlation coefficient (r_{xy}) and the coefficient of determination (R^2), respectively. As previously stated, for the GMR, $R_h^2 = R_E^2$. Furthermore, there is a simple relationship: $R_h^2 = R_E^2 = (1/2)(1+|r_{xy}|)$.[13]

Row (vi) reports standard errors (SE) of estimates $\hat{h}_i$ and $\hat{E}_i$. Row (viii) presents the wage elasticity of working hours (η), evaluated at the sample means of x and y. In the GMR, the wage elasticity is -0.170. The two wage elasticities by OLS regression (-0.205 and -0.139) are outside the standard error region of the GMR (-0.189 ~ -0.154). In addition, Column (4) presents the OLS regression estimate of x on y/x. It is known that using a wage rate variable measured by y/x results in division bias. The wage elasticity of

Table 1: Estimates of WH contract curve by GMR and OLS regression (14 manufacturing industries, male, college graduates, age 50–54, 2015)

		(1) GMR	*(2) OLS y on x*	*(3) OLS x on y*	*(4) OLS x on y/x*
(i)	Expression	E = α+ βh	E = α + βh	h = α + βE	h = α+ β(E/h)
(ii)	$\hat{\alpha}$ (Standard error)	3927.8 (400.0)	3245.8 (316.1)	199.2 (3.521)	197.0 (2.703)
(iii)	$\hat{\beta}$ (Standard error)	-19.02 (2.330)	-15.04 (1.840)	-0.0416 ($5.087*10^{-3}$)	-6.473 (0.651)
(iv)	r_{xy} (Correlation coefficient)	-0.791	-0.791	-0.791	-0.791
(v)	R^2 (Coefficient of determination)	0.895	0.626	0.626	0.712
(vi)	SE of estimate h	3.28		6.20	5.44
	SE of estimate E	62.4	118.0		
(vii)	Mean of sample x	171.5	171.5	171.5	171.5
	Mean of sample y	666.1	666.1	666.1	666.1
(viii)	Wage elasticity η (Region of standard error)	-0.170 (-0.189 ~ -0.154)	-0.205 (-0.227 ~ -0.187)	-0.139 (-0.153 ~ -0.124)	-0.147 (-0.161 ~ -0.132)
(ix)	Sample number	42	42	42	42

Note:
(a) Row (iv) is the correlation coefficient r_{xy} between x and y.
(b) Row (vi) represents the standard error (SE) of estimates h and E.
(c) Row (vii) denotes the sample mean of x and y.
(d) Row (viii) is the wage elasticity η, which is calculated by $w/(\hat{\beta}-w)$ (Columns 1 and 2), by $w\hat{\beta}/(1-w\hat{\beta})$ (Column 3), and by $w\hat{\beta}/x$ (Column 4). They are evaluated at the means of x and y, and w = y/x.

Data source: Basic Survey in Wage Structure (Ministry of Health, Labour and Welfare of Japan).

-0.147 (Column 4) lies outside the standard error region of GMR. These results indicate that we should not use OLS regression if the explanatory variables are measured with errors.[14]

3.4. GMR estimation results regarding four industry groups

We applied the same GMR analysis to the other three industry groups: transportation (eight industries), wholesale and retail (12 industries), and hotels *et al.* (six industries). We assumed that the industries in each group had similar production functions and that the workers in each group had similar preferences.

Table 2 presents the estimates for the four industry groups for 2015. The R^2 is at a passable level. In the groups of six, they are >8.0, and in the other two groups, they are >0.75.

Table 2: Estimates of the WH contract curve of four industry groups (male, age 50 54, 2015)

		College Graduates				*High School Graduates*			
		Manu-facturing	*Trans-portation*	*Wholesale and retail*	*Hotels et al.*	*Manu-facturing*	*Trans-portation*	*Wholesale and retail*	*Hotels et al.*
(i)	$\hat{\alpha}$	3927.8	3000.0	4865.1	2337.1	3114.8	2147.9	2424.1	1683.8
	(Standard error)	(400.0)	(511.3)	(934.8)	(359.4)	(515.1)	(395.2)	(455.6)	(505.5)
(ii)	$\hat{\beta}$	-19.02	-13.68	-24.84	-10.34	-14.39	-8.79	-10.94	-6.99
	(Standard error)	(2.330)	(2.908)	(5.462)	(1.996)	(2.844)	(2.097)	(2.545)	(2.713)
(iii)	r_{xy} (Correlation coefficient)	-0.791	-0.710	-0.616	-0.793	-0.626	-0.686	-0.595	-0.545
(iv)	R^2 (Coefficient of determination)	0.895	0.855	0.808	0.896	0.813	0.843	0.797	0.772
(v)	SE of estimate h	3.28	6.98	3.80	4.06	3.48	7.86	4.02	4.17
	Mean of sample x	171.5	175.3	171.1	179.8	181.0	187.9	178.9	186.3
(vi)	SE of estimate E	62.4	95.5	94.3	42.0	50.0	69.1	44.0	29.1
	Mean of sample y	666.1	601.0	616.3	476.8	510.3	495.3	466.7	381.9
(vii)	Wage Elasticity η	-0.170	-0.200	-0.127	-0.204	-0.164	-0.231	-0.193	-0.227
	(Region of standard error)	(-0.189~ -0.154)	(-0.241~ -0.171)	(-0.157~ -0.106)	(-0.241~ -0.173)	(-0.196~ -0.141)	(-0.282~ -0.195)	(-0.237~ -0.162)	(-0.324~ -0.174)
(viii)	Sample Number	42	24	36	18	42	22	36	18

Note:

(a) Expression of the estimated equation is $E = \alpha + \beta h$.

(b) Row (v) is the standard error (SE) of estimate h and mean of sample x.

(c) Row (vi) is the standard error (SE) of estimate E and mean of sample y.

(d) Row (vii) is wage elasticity η, calculated by $w/(\hat{\beta} - w)$ and evaluated at the means of x and y, and w = y/x. In parentheses is the region of the standard error.

Data source: Basic Survey in Wage Structure (Ministry of Health, Labour and Welfare of Japan).

The wage elasticities lie in the range of -0.13 and -0.23. The differences among the industry groups are not large; however, the transportation group has a slightly higher elasticity. Additionally, high school graduates have a slightly higher elasticity than college graduates.

4. Conclusion

The constructed model demonstrates that (1) working hours (h) and wage earnings (E) are determined jointly at the equilibrium point on a WH contract curve where the demand and supply of workers are equal, and (2) the WH contract curve passes through the intersection of the demand and supply curves of working hours. These results imply that (1) it is impossible to estimate the supply curve of working hours because equilibrium points are not usually located on a supply curve of working hours, and (2) the WH contract curve should be estimated to measure the wage elasticity of working hours.

For the estimation of the WH contract curves, GMR was selected for three reasons. First, both variables (h and E) in the contract curve equations are measured with errors. Second, the GMR estimator is the maximum likelihood estimator. Third, the coefficients of determination (R^2) of both variables (h and E) are equal in GMR. In GMR, there is a simple relation between the coefficient of determination (R^2) and the correlation coefficient (r_{xy}) as follows: $R^2 = (1 + |r_{xy}|)/2$.

The estimated wage elasticity of working hours for four industry groups is between -0.13 and -0.23. The transportation group has a slightly higher elasticity than the others, and high school graduates have a slightly higher elasticity than college graduates.

Notes

1. There are many survey articles on the supply curve of working hours. For example, see Heckman and Macurdy (1980), Killingsworth (1983), Pencavel (1986), Keane (2011) and Bargain and Peichl (2013).
2. In this model, we attempted to synthesise Lewis (1969), Rosen (1974), and Pencavel (2016).
3. An isoprofit curve is derived from the production function AF(L, h) as follows: If the output price is unity, the firm's profit is $\pi(L, h) = AF(L, h) - L\{E(h) + C\}$, where C denotes fixed costs per employee. As $\pi_L = \pi_h = 0$ on an isoprofit curve, we have the following two equations: $\pi_L = AF_L(L, h) - \{E(h)+C\} = 0$ and $\pi_h = AF_h(L, h) - LdE(h)/dh = 0$. Combining these two equations, we obtain $\{dE(h)/dh\}/\{E(h) + C\} = (1/L)\{F_h(L, h)/F_L(L, h)\}$. The solution $E = E(h)$ to this differential equation is the isoprofit curve. For example, when $F(L, h) = L^{\gamma}h^{\delta}$, the isoprofit curve equation is $E + C = kh^{(\delta/\gamma)}$. Here, k is the integral constant representing the profit level. For more details, refer to Kinoshita (1987, p.1274).

4. The WH contract curve is the locus of the tangency points of the isoprofit and indifference curves. We obtain the following Lagrangian equation: $\Gamma(E, h, \lambda) = E - \alpha(h + \beta)^2 + \lambda\{k - (E + C)/h^{(\delta/\gamma)}\}$, where λ denotes the Lagrangian multiplier. From the first-order condition, we obtain the following two equations: $\Gamma_E = 1 - \lambda h^{(-\delta/\gamma)} = 0$ and $\Gamma_h = -2\alpha(h + \beta) + \lambda(E+C)(\delta/\gamma)h^{(-\delta/\gamma-1)} = 0$. By eliminating λ from the equations, we obtain $(\delta/\gamma)(E + C) = 2\alpha h(h + \beta)$

5. Lewis (1969) describes the following market equilibrium of an industry: All workers have the same quality but different utility functions. All firms have different production functions. Then, the market equilibrium is not a point but a curve with a positive slope in the h-E plane. He calls this the "market equalizing wage curve." However, this curve provides little information on the supply or demand function for working hours. For further detail, refer to Lewis (1969) and Kinoshita (1987, p.1274).

6. As each industry consists of three parts based on the number of employees, the total sample number is 42.

7. The Deming regression is an errors-in-variables regression, named after Edwards Deming. The concept of the model was originally introduced in the late 1870s by Adcock and Kummel. This was revived by Koopmans (1937) and later propagated by Deming (1943). For more details, refer to Fuller (1987).

8. On this point, refer to Fuller (1987, p.31), Jensen (2007), and Gillard (2010).

9. Equation (7-1) includes cases of the OLS regression estimator. Solving (7-1) with respect to k, we have $k = \hat{\beta}(\hat{\beta}S_{xy} - S_{yy})/(S_{xy} - \hat{\beta}S_{xx})$. Therefore, if $k \to \infty$ (i.e., $var(e_{xi}) \to 0$), $\hat{\beta} \to S_{xy}/S_{xx}$, which is equal to the $\hat{\beta}$ of the OLS regression estimate of y on x. If k = 0 (i.e., $var(e_{yi}) = 0$), $\hat{\beta} = S_{yy}/S_{xy}$. This is the inverse of the OLS regression estimate of x on y. Additionally, $d\hat{\beta}/dk > 0$ is derived from the above equation.

10. From (7-2) and (7-4), we have $\hat{\beta}^2 = \dfrac{\Sigma(\hat{E}_i - \overline{y})^2}{\Sigma(\hat{h}_i - \overline{x})^2}$. In the GMR, from Equation (8), we have $\hat{\beta}^2 = \dfrac{\Sigma(y_i - \overline{y})^2}{\Sigma(x_i - \overline{x})^2}$. Therefore, $R_E^2 = \dfrac{\Sigma(\hat{E} - \overline{y})^2}{\Sigma(y_i - \overline{y})^2} = \hat{\beta}^2 \dfrac{\Sigma(\hat{h}_i - \overline{x})^2}{\Sigma(y_i - \overline{y})^2} = \dfrac{\Sigma(\hat{h}_i - \overline{x})^2}{\Sigma(x_i - \overline{x})^2} = R_h^2$.

11. In GMR, $k = \hat{\beta}^2$. Inserting this into (7-3), we have $\hat{h}_i = (1/2)[x_i + (1/\hat{\beta})(y_i - \hat{\alpha})] = (1/2)(BH+DH)$. Barker et al. (1988) presented the 'least triangular approach' to obtain a GMR estimator. This minimizes the sum of the triangular area BCD of all observed points.

12. (i) $\{S_{yy}/S_{xx}\} = \{S_{xy}/S_{xx}\}\{S_{yy}/S_{xy}\}$, which shows that Equation (8) is the geometric mean of the two corresponding OLS estimates.

    (ii) To calculate the standard errors of $\hat{\alpha}$ and $\hat{\beta}$ in GMR (Column 1), refer to Fuller (1987, pp.30-36).

13. From note (11), $\hat{h}_i = (y_i + \hat{\beta}x_i - \hat{\alpha})/2\hat{\beta}$; we insert this into the equation below:

$$= R_h^2 = \frac{\Sigma(\hat{h}_i - \overline{x})^2}{\Sigma(x_i - \overline{x})^2} = \frac{\Sigma\{(y_i + \hat{\beta}x_i - \hat{\alpha} - 2\hat{\beta}\overline{x})/2\hat{\beta}\}^2}{\Sigma(x_i - \overline{x})^2}$$

$$= \frac{1}{(2\hat{\beta})^2}\frac{\Sigma\{(y_i - \overline{y}) + \hat{\beta}(x_i - \overline{x})\}^2}{\Sigma(x_i - \overline{x})^2}$$

$$= 1/2 + \frac{1}{2\hat{\beta}}\frac{\Sigma(y_i - \overline{y})(x_i - \overline{x})}{\Sigma(x_i - \overline{x})^2}$$

$= (1/2)(1 + |r_{xy}|)$.

14. On division bias, refer to Borjas (1980).

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